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Abstracts

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ON CONVERGENCE ALMOST EVERYWHERE OF MULTIPLE TRIGONOMETRIC FOURIER SERIES

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Suppose d is a natural number, $\mathbb{T}^d = [-\pi, \pi)^d$ is a d-dimensional torus, $f \in L(\mathbb{T}^d)$. Let $\mathbf{n} = (n^1, n^2, \ldots, n^d)$ be a vector with nonnegative integer coordinates and let $S_{\mathbf{n}}(f, \mathbf{x})$ denote the $\mathbf{n}$th rectangular partial sum of the multiple trigonometric Fourier series of the function f. Let $\lambda \geq 1$. A multiple Fourier series of a function f is called λ-convergent at the point $\mathbf{x} \in \mathbb{T}^d$ if there exists a limit

$$\lim_{\min\{n^j : 1 \leq j \leq d\} \to +\infty} S_{\mathbf{n}}(f, \mathbf{x}),$$

considered only for those vectors $\mathbf{n} = (n^1, n^2, \ldots, n^d)$, where $1/\lambda \leq n^i/n^j \leq \lambda, \quad i, j \in \{1, \ldots, d\}$. λ-convergence is called convergence over cubes in the case $\lambda = 1$ and is called Pringsheim convergence in the case $\lambda = +\infty$.

It is well known that, if $L^p(\mathbb{T}^d)$, $p > 1$, $d \geq 2$, then the Fourier series of the function f converges over cubes almost everywhere. On the other hand, there exists an example of a continuous function of two variables, whose Fourier series λ-diverges everywhere on $\mathbb{T}^2$ for any $\lambda > 1$.

We consider one type of convergence of multiple trigonometric Fourier series intermediate between convergence over cubes and λ-convergence for $\lambda > 1$. The results about convergence and divergence almost everywhere of Fourier series in this sense will be discussed.

ON COMPARING SYSTEMS OF RANDOM VARIABLES WITH THE RADEMACHER SEQUENCE

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We ask whether inequalities between distributions of scalar polynomials of two sequences of random variables imply that the corresponding inequalities hold between the distributions of the norms of the corresponding vector sums in an arbitrary Banach space provided that one of the systems is the Rademacher system. We show that the answer is affirmative when the Rademacher functions form the majorizing system, and negative in the opposite case. We intend also to discuss connections of the above results published in [1] with the results of the paper [2].

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ON GREEDY APPROXIMATION WITH RESPECT TO AN ARBITRARY SET

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Let M be a subset of the Hilbert space H such that the metric projection $P_M(x)$ is nonempty for any $x \in H$. For every $x \in H$, the greedy algorithm with respect to M generates the sequence

$$x_0 = x, \quad x_{n+1} = x_n - y_n \quad (n = 0, 1, \dots),$$

where y_n is any element of $P_M(x_n)$.

We present several nontrivial conditions on M which are necessary or sufficient for convergence of this algorithm, that is, for $x_n \to 0$ independently of the initial element x.

CONSTRUCTIVE SPARSE APPROXIMATION WITH RESPECT TO THE FABER–SHAUDER SYSTEM

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This is joint work with Tino Ullrich (University of Bonn).

We consider approximations of multivariate functions using m terms from its tensorized Faber–Schauder expansion. The univariate Faber–Schauder system on $[0,1]$ is given by dyadic dilates and translates (in the wavelet sense) of the L_1 normalized simple hat function with support in $[0,1]$. We obtain a hierarchical basis which will be tensorized over all levels (hyperbolic) to get the dictionary $\mathcal{F}$. The worst-case error with respect to a class of functions $\mathbf{F} \hookrightarrow X$ is measured by the usual best m-term widths denoted by $\sigma_m(\mathbf{F}, \mathcal{F})_X$, where the error is measured in X. We constructively prove the following sharp asymptotical bound for the class of Besov spaces with small mixed smoothness (i.e. $1/p < r < \min\{1/\theta - 1, 2\}$) in L_q $(p < q \le \infty)$

$$\sigma_m(S^r_{p,\theta}B, \mathcal{F})_q \asymp m^{-r}$$

Note that this asymptotical rate of convergence does not depend on the dimension d (only the constants behind). In addition, this result holds for $q = 1$ and to our best knowledge this is the first sharp result involving L_1 as a target space. We emphasize two more things. First, the selection procedure for the coeffcients is a level-wise constructive greedy strategy which only touches a finite prescribed number of coeffcients. And second, due to the use of the Faber-Schauder system, the coeffcients are finite linear combinations of discrete Function values. Hence, this method can be considered as a nonlinear adaptive sampling algorithm leading to a pure polynomial rate of convergence for any d.

NEW RESULTS CONNECTED WITH HARDEY–LITTLEWOOD–PALEY THEOREM

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(joint results with E.D. Nursultanov and S.Yu. Tikhonov)

In this talk, 3 different topics will be discussed.
1. The possibility of extending the areas of p in classical Hardy–Littlewood–Paley result for transformed Fourier series.
2. The behavior of the constants in classical Hardy–Littlewood–Paley result for transformed Fourier series.
3. The extension of Hardy–Littlewood result for trigonometric series with monotone coefficients to the series with generale monotone coefficients.

NIKOLSKII CONSTANTS FOR SPHERICAL POLYNOMIALS AND ENTIRE FUNCTIONS OF SPHERICAL EXPONENTIAL TYPE

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(joint results with F. Dai and S. Tikhonov)

We study the asymptotic behavior of sharp Nikolskii constant

$$\mathcal{C}(n,d,p,q) := \sup\{\|f\|_{L^q(\mathbb{S}^d)} \colon f\in\Pi_n^d,\ \|f\|_{L^p(\mathbb{S}^d)}=1\}$$

for $0<p<q\le\infty$ as $n\to\infty$, where Π_n^d denotes the space of all spherical polynomials f of degree at most n on the unit sphere $\mathbb{S}^d\subset\mathbb{R}^{d+1}$.

1. We prove that for $0<p<\infty$ and $q=\infty$,

$$\lim_{n\to\infty}\frac{\mathcal{C}(n,d,p,\infty)}{n^{d/p}}=\mathcal{L}(d,p,\infty),$$

and for $0<p<q<\infty$,

$$\liminf_{n\to\infty}\frac{\mathcal{C}(n,d,p,q)}{n^{d\,(1/p-1/q)}}\ge\mathcal{L}(d,p,q),$$

where the constant $\mathcal{L}(d,p,q)$ is defined for $0<p<q\le\infty$ by

$$\mathcal{L}(d,p,q) := \sup\{\|f\|_{L^q(\mathbb{R}^d)} \colon f\in\mathcal{E}_p^d,\ \|f\|_{L^p(\mathbb{R}^d)}=1\},$$

with $\mathcal{E}_p^d$ denoting the set of all entire functions $f\in L^p(\mathbb{R}^d)$ of spherical exponential type at most 1.

These results extend the recent results of Levin and Lubinsky for trigonometric polynomials on the unit circle.

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Compared with those in one variable, our proof in higher-dimensional case is more difficult because functions on the sphere can not be identified as periodic functions on Euclidean space and explicit connections between spherical polynomial interpolation and the Shannon sampling theorem for entire functions of exponential type are not available.

Our proof of the upper estimate relies on a recent deep result of Bondarenko, Radchenko and Viazovska on spherical designs:

$$\frac{1}{|\mathbb{S}^d|}\int_{\mathbb{S}^d} f(x)\,dx = \frac{1}{N}\sum_{j=1}^{N} f(x_{n,j}), \quad f\in\Pi_n^d,$$

an earlier result of Yudin on the distribution of points of spherical designs $\{x_{n,j}\}$, and also our previous result on a connection between positive cubature formulas and the Marcinkiewitcz–Zygmund inequality on the sphere:

$$\|f\|_p \asymp \Big(\sum_{\omega\in\Lambda}\lambda_{n,j}|f(x_{n,j})|^p\Big)^{1/p}, \quad 0<p<\infty.$$

The proof of the lower estimate is based on the de la Vallée-Poussin type kernels associated with a smooth cutoff function on the sphere and also some properties of the exponential mapping from the tangent plane to the sphere, which connects functions on sphere with functions on Euclidean space.

2. While it remains a very challenging open problem to determine the exact value of the Nikolskii constant $\mathcal{L}(d,1,\infty)$, we are able to find the exact value of the Nikolskii constant for $p=1$, $q=\infty$ and nonnegative functions $f\in\mathcal{E}_1^d$:

$$\sup_{0\le f\in\mathcal{E}_1^d}\frac{\|f\|_{L^\infty(\mathbb{R}^d)}}{\|f\|_{L^1(\mathbb{R}^d)}} = \frac{1}{2^{d-1}|\mathbb{S}^d|\Gamma(d+1)}.$$

3. We investigate the normalized Nikolskii constant

$$L_d := \frac{|\mathbb{S}^d|\Gamma(d+1)}{2}\,\mathcal{L}(d,1,\infty).$$

For this problem, we first show existence of an extremal function and its uniqueness.

It was known that $L_d\le 1$ and $L_d\ge e^{-d(1+o(1))}$ as $d\to\infty$. We improve these bounds as follows:

$$2^{-d}\le L_d\le {}_1F_2\Big(\frac{d}{2};\frac{d}{2}+1,\frac{d}{2}+1;-\frac{\beta_d^2}{4}\Big),$$

where ${}_1F_2$ and β_d denote the hypergeometric function and the smallest positive zero of the Bessel function $J_{d/2}$, respectively. This implies that the constant L_d decays exponentially fast as $d\to\infty$:

$$(0.5)^d\le L_d\le (\sqrt{2/e})^{d(1+o(1))}, \quad \sqrt{2/e}=0.85776\cdots.$$

4. We observe that for $d\ge 2$, the asymptotic order of the usual Nikolskii inequality on $\mathbb{S}^d$ can be significantly improved in many cases, for lacunary spherical polynomials of the form $f=\sum_{j=0}^m f_{n_j}$ with f_{n_j} being a spherical harmonic of degree n_j and $n_{j+1}-n_j\ge 3$. As is well known, for $d=1$, the Nikolskii inequality for trigonometric polynomials on the unit circle does not have such a phenomenon.

DISCRETIZATION OF L_p-NORMS OF ENTIRE FUNCTIONS OF EXPONENTIAL TYPE WITH WEIGHT

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(joint results with D.V. Gorbachev and S.Yu. Tikhonov)

The talk will be devoted to the discussion of bilateral discrete estimates of the L_p-norms of entire functions of exponential type. We will give well-known results of Plancherel–Polya, Boas, Nikol'skii, Pesenson and others in the spaces $Lp(\mathbb{R}^d)$, as well as our results in $Lp(\mathbb{R}^d)$ with Dunkl type weight.

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REGULARITY SPACES AND WAVELET IN A GEOMETRIC FRAMEWORK. APPLICATION TO GAUSSIAN PROCESSES AND STATISTICAL ESTIMATION

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We will show how, on a suitable Dirichlet space, one can define regularity spaces, and a wavelet system. This construction works for Riemannian manifolds, under some curvature condition. Then we revisit some classical results on the behavior of Gaussian processes, and non-parametric adaptive statistical estimation.

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UNCONDITIONAL CONVERGENCE FOR WAVELET FRAME EXPANSIONS

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We study unconditional convergence for wavelet frame expansions in $L_p(\mathbb{R})$.

Let $\{\psi_{j,k}\}_{(j,k)\in\mathbb{Z}^2}$, $\{\tilde{\psi}_{j,k}\}_{(j,k)\in\mathbb{Z}^2}$ be dual wavelet frames in $L_2(\mathbb{R})$, let η be an even, bounded, decreasing on $[0,\,\infty)$ function such that $\int\limits_0^\infty \eta(x)\ln(1+x)\,dx<\infty$, and $|\psi(x)|, |\tilde{\psi}(x)|\le\eta(x)$. Than the series $\sum_{j,k\in\mathbb{Z}}(f,\,\tilde{\psi}_{j,k})\psi_{j,k}$ is unconditional convergent in $L_p(\mathbb{R})$, $1<p<\infty$.

FAST DISCRETE FOURIER TRANSFORM ON LOCAL FIELDS OF POSITIVE CHARACTERISTIC

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Let $K=F(s)$ be a local field of positive characteristic p over a finite field $GF(p^s)$, K^+ the additive group of the field K.

The local field $F(s)$ is a set of infinite sequences $a=(...,0,\mathbf{a}_n,\mathbf{a}_{n+1})$ of elements $\mathbf{a}_j=(a_j^0,a_j^1,...,a_j^{p-1})$ from the finite field $GF(p^s)$. The base of topology consists from all subgroups $K_n^+=\{a=(...,0,\mathbf{a}_n,\mathbf{a}_{n+1},...)\}$.

Elements $g_n=(...,0_{n-1},(1,0...0)_n,0_{n+1},...)$ form a bases of additive group K^+. The set of step functions constant on cosets of the subgroup K_M^+ with the support $\mathrm{supp}(\varphi)\subset K_{-N}^+$ will be denoted as $\mathfrak{D}_M(K_{-N}^+)$, $M,N\in\mathbb{Z}$.

Any additive character χ may be written in the form $\chi=\prod_{k\in\mathbb{Z}}\mathbf{r}_k^{\mathbf{a}_k}$. where

$$r_{ks+0}^{a_k^{(0)}}r_{ks+1}^{a_k^{(1)}}\dots r_{ks+s-1}^{a_k^{(s-1)}}=\mathbf{r}_k^{\mathbf{a}_k}$$

and $\mathbf{a}_k=(a_k^{(0)},a_k^{(1)},\dots,a_k^{(s-1)})\in GF(p^s)$. We will refer to $\mathbf{r}_k^{(1,0,\dots,0)}=\mathbf{r}_k$ as the Rademacher functions. The definition of Rademacher function implies that if $\mathbf{x}=((x_k^{(0)},x_k^{(1)},\dots x_k^{(s-1)}))_{k\in\mathbb{Z}}$ and
$\mathbf{u}=(u^{(0)},u^{(1)},\dots,u^{(s-1)})\in GF(p^s)$ then $(\mathbf{r}_k^{\mathbf{u}},\mathbf{x})=\prod\limits_{l=0}^{s-1}e^{\frac{2\pi i}{p}u^{(l)}x_k^{(l)}}$.

Theorem. *If $f^{(N)}\in\mathfrak{D}_N(K_0^+)$ then*

$$f^{(N)}=\sum_{\bar{\alpha}_0\bar{\alpha}_1\dots\bar{\alpha}_{N-1}\in GF(p^s)}c_{\bar{\alpha}_0,\bar{\alpha}_1,\dots,\bar{\alpha}_{N-1}}\mathbf{r}_0^{\bar{\alpha}_0}\mathbf{r}_1^{\bar{\alpha}_1}\dots\mathbf{r}_{N-1}^{\bar{\alpha}_{N-1}}. \quad (1)$$

Let us denote

$$f^{(N)}_{\mathbf{a}_{N-1}\mathbf{a}_{N-2}\dots\mathbf{a}_1\mathbf{a}_0}:=f^{(N)}(K_N\dot{+}\mathbf{a}_{N-1}g_{N-1}\dot{+}\dots\dot{+}\mathbf{a}_1g_1\dot{+}\mathbf{a}_0g_0).$$

[1]Supported by the Russian Foundation for Basic Research, project no. 16-01-00152.

So, the Fourier transform takes the vector of values $f^{(N)}_{\mathbf{a}_{N-1}\mathbf{a}_{N-2}\ldots\mathbf{a}_1\mathbf{a}_0}$ into the vector of coefficients $c_{\bar{\alpha}_0,\bar{\alpha}_1,\ldots,\bar{\alpha}_{N-1}}$

Using the theorem 1 we obtain for $N+1 \geq n \geq 1$ recursion relations

$$f^{(n)}_{(\bar{\alpha}_N\ldots\bar{\alpha}_n)\mathbf{a}_{n-1}\ldots\mathbf{a}_1\mathbf{a}_0} = \sum_{\bar{\alpha}_{n-1}\in GF(p^s)} p^{\frac{-s}{2}} e^{\frac{2\pi i}{p}(\bar{\alpha}_{n-1},\mathbf{a}_{n-1})} p^{\frac{s}{2}} f^{(n-1)}_{(\bar{\alpha}_N,\ldots\bar{\alpha}_{n-1})\mathbf{a}_{n-2}\ldots\mathbf{a}_1\mathbf{a}_0}. \tag{2}$$

It is calculate formula for inverse Fourier transform. For $n=1$ we obtain equalities

$$f^{(1)}_{(\bar{\alpha}_N,\bar{\alpha}_{N-1},\ldots,\bar{\alpha}_1)\mathbf{a}_0} = \sum_{\bar{\alpha}_0\in GF(p^s)} e^{\frac{2\pi i}{p}(\bar{\alpha}_0,\mathbf{a}_0)} f^{(0)}_{(\bar{\alpha}_N,\bar{\alpha}_{N-1},\ldots,\bar{\alpha}_1\bar{\alpha}_0)},$$

from which we find the Fourier coefficients $c_{\bar{\alpha}_0\bar{\alpha}_1\ldots\bar{\alpha}_N} = f^{(0)}_{(\bar{\alpha}_N,\bar{\alpha}_{N-1},\ldots,\bar{\alpha}_1\bar{\alpha}_0)}$. Solving system (2) we obtain the solution

$$f^{(n-1)}_{(\bar{\alpha}_N\ldots\bar{\alpha}_{n-1})\mathbf{a}_{n-2}\ldots\mathbf{a}_1\mathbf{a}_0} = \sum_{\mathbf{a}_{n-1}\in GF(p^s)} p^{-s} e^{-\frac{2\pi i}{p}(\bar{\alpha}_{n-1},\mathbf{a}_{n-1})} f^{(n)}_{(\bar{\alpha}_N,\ldots\bar{\alpha}_n)\mathbf{a}_{n-1}\ldots\mathbf{a}_1\mathbf{a}_0}, \tag{3}$$

Formulas (2) and (3) are the inverse and direct Fourier transform respectively. Fourier transform on the field $F(s)$ is an analog of s-dimensional Fourier transform. The number of operations of both direct and inverse Fourier transform is equal to $(N+1)\cdot 2p^{2s}\cdot p^{Ns} = 2(N+1)p^{s(N+2)}$.

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AROUND THE UNCERTAINTY PRINCIPLE

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How "small" can be the support and the spectrum of L^2 function?
I'll discuss some background and present a recent result joint with Fedor Nazarov.

VILENKIN–CHRESTENSON SERIES AND RECOVERY OF SUMMABLE FUNCTIONS

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Let $\mathcal{P}$ be any sequences of primes, $G = G_{\mathcal{P}}$ be the $\mathcal{P}$-adic group, $\{\gamma_n\}$ be the system of characters of G in the Paley enumeration (Vilenkin–Chrestenson(–Paley) system), and μ be the normalized Haar measure on G (see, for example, [1–3]).

Consider series $\sum_{n=0}^{\infty} a_n\gamma_n(g)$. Then there exist a countable set $Q \subset G$, open sets $E_k \subset G$, and sets $N_k \subset \mathbb{N}$, $k = 0, 1, \ldots$, with the following properties:

1) $\mu(E) < 1$ where $E = \bigcup_{k=0}^{\infty} E_k$;
2) $Q \subset E$;
3) $\max\{n \in N_k\} < \min\{n \in N_{k+1}\}$ for all k;
4) almost all the values of any function $f \in L(G)$ are recovered if we know the means of f on each portion of E_k and all the values

$$\sum_{n \in N_k} a_n\gamma_n(g), \qquad g \in Q.$$

Some corollaries from this result will be discussed.

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MULTIVARIATE HAAR FUNCTIONS, BOUNDARY DIMENSION, AND SYNCHRONIZING AUTOMATA

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Multivariate Haar functions are constructed on R^d with an arbitrary integer dilation expansive $d \times d$ matrix M with a certain set of $m = |\det M|$ "digits" from the corresponding quotient subsets of Z^d. Unlike the univariate case, there are many different Haar systems on R^d, they may have various smoothness depending on the matrix M and on the set of digits. Computation of their Holder regularity in L_2 is notoriously hard problem.

A formula for the Hölder exponent in case of general dilation matrices was recently obtained in [1]. We show that the L_2 Hölder exponent of a Haar function can be expressed by the boundary dimension of its support G. This is an analogue to the Hausdorff dimension of the boundary, but do not coincide with it. Moreover, the same value has a rather surprising interpretation in terms of the problem of synchronizing automata. A finite automata is determined by a directed multigraph with N vertices (states) and with all edges (transfers) coloured with m colours so that each vertex has precisely one outgoing edge of each colour. The automata is synchronizing if there exists a finite sequence of colours such that all paths following that sequence terminate at the same vertex independently of the starting vertex. The problem of synchronizing automata has been studied in great detail (see [2] for a survey). It turns out that each Haar function can be naturally associated with a finite automata and the Hölder exponent is related to the length of the synchronizing sequence. We introduce a concept of synchronizing rate and show that it is actually equal to the Hölder exponent of the corresponding Haar function. Applying this result we prove that the Hölder exponent can be found within finite time by a combinatorial algorithm.

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SOME PROPERTIES OF THE SPACE OF QUASICONTINUOUS FUNCTIONS

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For $f \in L^p(0, 2\pi)$ we set

$$\|f\|_p = \left(\int_0^{2\pi} |f(x)|^p \, dx \right)^{1/p} \qquad \text{for } 1 \le p < \infty,$$
$$\|f\|_\infty = \operatorname*{ess\,sup}_{[0,2\pi]} |f(x)| \qquad \text{for } p = \infty.$$

Let n be a natural number. Denote by E_n the space of real trigonometric polynomials of the form

$$t(x) = \sum_{k=2^{n-1}}^{2^n-1} a_k \cos kx + b_k \sin kx.$$

For $f \in L^1(0, 2\pi)$ with the Fourier series $f \sim \sum_{j=0}^{\infty} \delta_j(f, x)$, where $\delta_0(f) = a_0(f)/2$,

$$\delta_j(f, x) = \sum_{k=2^{j-1}}^{2^j-1} a_k(f) \cos kx + b_k(f) \sin kx, \quad j = 1, 2, \ldots,$$

we put

$$\|f\|_{\mathrm{QC}} \equiv \int_0^1 \Big\| \sum_{j=0}^{\infty} r_j(t) \delta_j(f, \cdot) \Big\|_\infty dt, \tag{1}$$

where $\{r_j(t)\}_{j=0}^{\infty}$ is the Rademacher system and $a_k(f)$ and $b_k(f)$ are Fourier coefficients of the function f. *The space of quasicontinuous functions* is defined as the closure of trigonometric polynomials in norm (1). The space of quasicontinuous functions and QC-norm were introduced by Kashin and Temlyakov (see [1]).

We prove the following result (see [2]).

THEOREM. *Let $\varepsilon \in (0, 1)$ be a real number, let $1 \le k_1 < k_2 < \ldots < k_n < \ldots$ be a sequence of natural numbers, let L_n be a subspace of E_{k_n} such that* $\dim L_n \ge \varepsilon \dim E_{k_n}$, $n = 1, 2, \ldots$. *Then*

$$\sup_{0 \ne t \in L_1 \oplus \cdots \oplus L_n} \frac{\|t\|_{\mathrm{QC}}}{\|t\|_\infty} \ge c(\varepsilon)\sqrt{n}, \quad n = 1, 2, \ldots,$$

where $c(\varepsilon) > 0$ is a constant which depends only on ε.

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OPTIMAL SAMPLING RATES FOR APPROXIMATING AN ANALYTIC FUNCTION ON A COMPACT

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We consider the problem of approximating an analytic function on a compact interval from its values at $M + 1$ distinct points. When the points are equispaced, a recent result (the so-called impossibility theorem) has shown that the best possible convergence rate of a stable method is root-exponential in M, and that any method with faster exponential convergence must also be exponentially ill-conditioned at a certain rate.

Here, we present an extension of the impossibility theorem valid for general nonequispaced points, and apply it to the case of points that are equidistributed with respect to modified Jacobi measures. This leads to a necessary sampling rate for stable approximation from such points. We prove that this rate is also sufficient, and therefore exactly quantify (up to constants) the precise sampling rate for approximating analytic functions from such node distributions with stable numerical methods.

In particular, we theoretically confirm the well-known heuristic that stable least-squares approximation using polynomials of degree $N < M$ is possible only once M is sufficiently large for there to be a subset of N of the nodes that mimic the behaviour of the Nth Chebyshev nodes.

APPROXIMATION BY MULTIVARIATE QUASI-PROJECTION OPERATORS

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(joint results with A. Krivoshein and Yu. Kolomoitsev)

Quasi-projection operators with a matrix dilation M are

$$Q_j(f, \phi, \widetilde{\phi}) = \sum_{k \in \mathbb{Z}^d} \langle f, \widetilde{\phi}_{jk} \rangle \phi_{jk},$$

where ϕ is a function, and $\widetilde{\phi}$ is a function or a tempered distribution,

$$\psi_{jk}(x) := m^{j/2} \psi(M^j x + k), \quad j \in \mathbb{Z}, k \in \mathbb{Z}^d,$$

M is a $d \times d$ matrix whose eigenvalues are bigger than 1 (in absolute value), $m = |\det M|$.

We consider different classes of such operators and study their approximation properties. Error estimates in L_p-norm, $2 \le p \le \infty$, are provided for a large class of functions ϕ (including both band-limited and compactly supported functions) and for $\widetilde{\phi} \in \mathcal{S}'_N$, where $\mathcal{S}'_N$ is the set of tempered distribution whose Fourier transform $\widehat{\widetilde{\phi}}$ is a function on $\mathbb{R}^d$ such that $|\widehat{\widetilde{\phi}}(\xi)| \le C_{\widetilde{\phi}} |\xi|^N$ for almost all $\xi \notin \mathbb{T}^d$, $N = N(\widetilde{\phi}) \ge 0$, and $|\widehat{\widetilde{\phi}}(\xi)| \le C'_{\phi}$ for almost all $\xi \in \mathbb{T}^d$. The estimates are given in terms of the Fourier transform of f. In particular, a finite linear combination of the Dirac delta-function and its derivatives is in $\mathcal{S}'_N$. If $\widetilde{\phi}$ is the Dirac delta-function and $\phi = \mathrm{sinc}$, then $Q_j(f, \phi, \widetilde{\phi})$ is the classical sampling operator.

Another class of quasi-projection operators we study includes classical Kantorovich–Kotelnikov operators, where $\widetilde{\phi}$ is the characteristic function of $[0, 1]$. In this case $\langle f, \widetilde{\phi}_{jk} \rangle$ is the averages value of f near the node $M^{-j}k$ (instead of the exact value $f(M^{-j}k)$ in the sampling expansion), which allows to deal with discontinues signals and reduce the so-called time-jitter errors. Error estimates in L_p-norm, $1 \le p \le \infty$, for this class are given in terms of classical moduli of smoothness. Such estimates are aimed at the recovery of signals f, but they are not applicable to non-decaying signals and for signals whose decay is not enough to be in L_p, which are of interest to engineers. However such signals may belong to a weighted L_p space. Error estimates in the weighted L_p spaces are also obtained for the Kantorovich–Kotelnikov-type and sampling operators.

BOUNDS FOR L_p-DISCREPANCIES OF POINT DISTRIBUTIONS IN COMPACT METRIC MEASURE SPACES

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It will be shown in the talk that nontrivial upper bounds for the L_p-discrepancies of point distributions in compact metric measure spaces can be proved for all exponents $0 < p < \infty$ and $p = \infty$ under very simple conditions on the volume of metric balls as a function of radii. Particularly, these conditions hold for all compact Riemannian manifolds. Such upper bounds are sharp, at least, for $2 \leq p < \infty$ and Riemannian symmetric manifolds of rank one. (The paper with the detailed proofs is available at arXiv: 1802.01577).

ESTIMATES FOR THE SUMS OF A SINE SERIES WITH CONVEX COEFFICIENTS

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Let's consider the sum of a sine series $g(\mathbf{b}, x) = \sum_{k=1}^{\infty} b_k \sin kx$. It is known that the sum of a sine series with convex coefficients $\mathbf{b} = \{b_k\}_{k\in\mathbb{N}}$ is positive in the interval $x \in (0, \pi)$. To estimate values of the sum near the origin traditionally was used function introduced by Salem $v(\mathbf{b}, x) = x \sum_{k=1}^{m(x)} k b_k$, $m(x) = [\pi/x]$. S. A. Telyakovskiĭ [1] and A. Yu. Popov [2] proved the inequality $g(\mathbf{b}, x) > 2\pi^{-2} v(\mathbf{b}, x) + o(1)$, $x \to +0$, with exact constant "$2\pi^{-2}$". We show that the function $2\pi^{-2} v(\mathbf{b}, x)$ is not a minorant for $g(\mathbf{b}, x)$. It is proved that for the modified Salem function $v_0(\mathbf{b}, x) = x \left(\sum_{k=1}^{m(x)-1} k b_k + (1/2) m(x) b_{m(x)} \right)$ the lower estimate $g(\mathbf{b}, x) > 2\pi^{-2} v_0(\mathbf{b}, x)$ is true in some right neighborhood of zero. It is established that this estimate is sharp in the class of convex sequences $\mathbf{b}$.

Theorem 1. *Let $\mathbf{b}$ be positive convex null sequence. Then for some $x_0 > 0$ the following inequality holds*

$$g(\mathbf{b}, x) > \frac{2}{\pi^2} v_0(\mathbf{b}, x), \qquad 0 < x < x_0.$$

For every $\varepsilon > 0$ there exists a convex slowly varying null sequence $\mathbf{b}$, for which there is a sequence of points $\{x_n\}_{n=1}^{\infty}$, $x_n \to +0$, such that

$$g(\mathbf{b}, x_n) < \frac{2}{\pi^2} x_n \left(\sum_{k=1}^{m(x_n)-1} k b_k + \left(\frac{1}{2} + \varepsilon \right) m(x_n) b_{m(x_n)} \right).$$

This result is most interesting when the sequence of coefficients $\mathbf{b}$ is slowly varying. In this case the following asymptotic formula holds:

$$\frac{2}{\pi^2} v(\mathbf{b}, x) \sim \frac{b_{m(x)}}{x}, \qquad x \to +0.$$

Aljančić, Bojanić and Tomić established [3], that for any convex slowly varying null sequence $\mathbf{b}$ the following asymptotic formula holds:

$$g(\mathbf{b}, x) \sim \frac{b_{m(x)}}{x}, \qquad x \to 0.$$

We refine marked result. It is shown that the difference

$$g(\mathbf{b}, x) - \frac{b_{m(x)}}{x}$$

in order is comparable with the function

$$\sigma(\mathbf{b}, x) = x \sum_{k=1}^{m(x)} \frac{k(k+1)}{2}(b_k - b_{k+1}).$$

A two-sided estimate of this difference with sharp constants is obtained.

THEOREM 2. *Let $\mathbf{b}$ be positive convex slowly varying null sequence. Then for some $x_0 > 0$ the following inequalities hold:*

$$\frac{b_{m(x)}}{x} + \frac{6(\pi-1)}{\pi^3}\sigma(\mathbf{b}, x) - \frac{b_{m(x)} - b_{m(x)+1}}{x} < g(\mathbf{b}, x) < \frac{b_{m(x)}}{x} + \sigma(\mathbf{b}, x),$$

where $0 < x < x_0$.

There are convex slowly varying null sequences $\underline{\mathbf{b}}$, $\overline{\mathbf{b}}$, such that

$$\varliminf_{x\to+0} \left(g(\underline{\mathbf{b}}, x) - \frac{b_{m(x)}}{x} \right) / \sigma(\underline{\mathbf{b}}, x) = \frac{6(\pi-1)}{\pi^3},$$

$$\varlimsup_{x\to+0} \left(g(\overline{\mathbf{b}}, x) - \frac{b_{m(x)}}{x} \right) / \sigma(\overline{\mathbf{b}}, x) = 1.$$

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3. S. Aljančić, R. Bojanić and M. Tomić. *Sur le comportement asymptotique au voisinage de zéro des séries trigonométriques de sinus à coefficients monotones*, Acad. Serbe Sci. Publ. Inst. Math., 1956, 10, 101–120.

BANACH FRAMES, REPRODUCING KERNELS AND DISCRETIZATION

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Let X be a Banach space and let X_d be a sequence Banach space with a natural basis.

We say that a sequence $\{x_n\}_{n=1}^{\infty} \subset X \setminus \{0\}$ of elements of a Banach space X is a frame for X with respect to X_d if there exist constants $0 < A \le B < \infty$ such that for all bounded linear functionals $x^* \in X^*$ the following inequalities are satisfied

$$A\|x^*\|_{X^*} \le \|\{\langle x_n, x^*\rangle\}_{n=1}^{\infty}\|_{X_d^*} \le B\|x^*\|_{X^*}.$$

Let

$$\widehat{K}_{\lambda_n}(z) = \frac{(1-|\lambda_n|^2)^{\frac{1}{2}}}{1-\overline{\lambda_n}z}, \qquad z \in \mathbb{D}, \qquad n = 1, 2, \ldots, \tag{1}$$

denote values of the normalized reproducing Szegö kernel $K(z,\lambda) = (1-\overline{\lambda}z)^{-1}$ for the Hardy space $H^2(\mathbb{D})$.

It is well-known that sequence (1) is not a basis for the Hardy space $H^2(\mathbb{D})$ for any set of points $\{\lambda_n\}_{n=1}^{\infty}$. Moreover, sequence (1) can not be a Duffin-Schaeffer frame for $H^2(\mathbb{D})$. Nevertheless, there are points

$$\lambda_n = \lambda_{k,j} = r_k e^{\frac{2\pi i j}{k}}, \qquad j = 0, \ldots, n_k - 1, \qquad k = 1, 2, \ldots,$$

such that sequence (1) fulfills the frame inequalities

$$A\|f\|_{H^2} \le \sup_{k=1,2,\ldots} \left(\sum_{j=0}^{n_k-1} |\langle f, \widehat{K}_{\lambda_{k,j}}\rangle|^2 \right)^{\frac{1}{2}} \le B\|f\|_{H^2},$$

which are equivalent to the discretization inequalities

$$A'\|f\|_{H^2} \le \sup_{k=1,2,\ldots} \left(\frac{1}{n_k} \sum_{j=0}^{n_k-1} |f(\lambda_{k,j})|^2 \right)^{\frac{1}{2}} \le B'\|f\|_{H^2}$$

when the next matching conditions are satisfied

$$\frac{a}{n_k} \le 1 - r_k \le \frac{b}{n_k}, \qquad k = 1, 2, \ldots.$$

It follows that for every $f \in H^2(\mathbb{D})$ there exist coefficients $c_n = c_{k,j}$ such that

$$\sum_{k=1}^{\infty} \left(\sum_{j=0}^{n_k-1} |c_{k,j}|^2 \right)^{\frac{1}{2}} < \infty$$

and the representation is valid

$$f = \sum_{n=1}^{\infty} c_n \widehat{K}_{\lambda_n}.$$

CHEBYSHEV PROJECTION IN NON-SYMMETRICAL POLYHEDRAL SPACES

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Let X be a finite dimensional linear space and let $A \subset X$ be a convex bounded polyhedron and its interior consists zero of X. We consider on X a non-symmetrical norm $\|\cdot|$ generated by the functional Minkowski of the set A. The pair $(X, \|\cdot|)$ is said to be non-symmetrical polyhedral space. A set $V \subset X$ is said to be a linear polyhedral set if it is a finite intersection of some closed half spaces of X. Let $B(x,r)$ be a ball with center x and radius $r \geq 0$. A set $M \subset X$ is said bounded if there exists a ball $B(x,r) \supset M$. A quantity $r_V(M) := \inf\{r \geq 0 \mid M \subset B(x,r),\, x \in V\}$ is said to be relative (relatevly of the set V) Chebyshev radius of M and any point $x \in V$ is said to be relative Chebyshev center (relatevly of the set V) if $M \subset B(x, r_V(M))$. We denote by $Z_V(\cdot)$ the Chebyshev projection which by definition any bounded set corresponds to the sets of all its relative Chebyshev centers. For arbitraries bounded sets $M, N \subset X$ we denote by $h(M,N)$ the Hausdorff distance between of these sets. We note that contraction on class single-point sets the Chebyshev projection is represented the metric projection which by definition corresponds any point of X to their sets of all nearest points of V. In particularly if $V = X$ the Chebyshev projection denoted by $Z(\cdot)$ and any bounded set corresponds to the sets of all its Chebyshev centers. Druzhinin proved that there exists a Lipschitz selection from the operator $Z(\cdot)$ in any symmetrical polyhedral finite dimensional space. W. Li и W. Li, M. Finzel established that the metric projection on arbitrary linear polyhedral set of finite dimensional non-symmetrical polyhedral space is a multi-valued Lipschitz map which admits a Lipschitz selection from itself. The next assertion are generalized above results.

Theorem 1. *Let $(X, \|\cdot|)$ be a non-symmetrical finite dimensional polyhedral space and let $V \subset X$ be a nonempty linear polyhedral set. Then there exist a number $c = c(V) > 0$ that for any bounded sets $M, N \subset X$ the estimate $h(Z_V(M), Z_V(N)) \leq ch(M,N)$ holds and there exist a single-valued Lipschitz map $M \overset{\varphi}{\to} Z_V(M)$ (i.e. a selection from the operator $Z_V(\cdot)$) which corresponds any bounded set to a its relative Chebyshev center.*

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INTEGRATION OF SMOOTH FUNCTIONS USING RANDOM POINT SETS

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(joint results with M. Griebel, A. Hinrichs, J. Oettershagen)

We study numerical integration of multivariate smooth functions f from a given class

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of functions F by cubature rules of the form

$$Q^{\mathbf{w}}_{\mathcal{P}_n}(f) = \sum_{i=1}^{n} w_i f(x_i)$$

where $\mathcal{P}_n = \{x_1, \ldots, x_n\}$ with $x_i \in [0,1]^d$ and $\mathbf{w} = (w_1, \ldots, w_n) \in \mathbb{R}^n$.

For given weights, e.g. $w_1 = \cdots = w_n = 1/n$, there are a lot of results concerning good choices for point sets $\mathcal{P}_n$ such that this method satisfies good or even optimal worst-case error bounds in specific classes of functions F. However, most of the known explicit constructions are very hard to implement in high dimensions, even if one considers classes of smooth functions. Here, we consider random point sets $\mathcal{P}_n$, where the points are independent and uniformly distributed in $[0,1]^d$, which is a cheap 'construction' also in very high dimensions. We show that, with high probability, we can choose the weights $\mathbf{w}$ such that $Q^{\mathbf{w}}_{\mathcal{P}_n}$ has (up to logarithmic factors) the optimal order of convergence in several Sobolev and Besov spaces of dominating mixed smoothness. Moreover, the algorithm for choosing the weights is computationally easy and independent of the specific space which makes the cubature rule (to some extent) universal.

We show numerical results that suggest the efficiency of this algorithm.

LOW-RANK APPROXIMATION OF NEAREST NEIGHBOR INTERACTION SYSTEMS

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Low-rank tensor methods are an important tool for the numerical treatment of equations with a high-dimensional state space. Nearest neighbor interaction systems like the Ising model or some Chemical Master equations are examples for this, and the low-rank tensor train format has shown to be efficient for their computation in some cases. A challenging task, however, is to provide theoretical justification for this. For ground states of 1D quantum spin systems such arguments have been found in the theoretical physics community, but they can be applied more generally. The idea is to study the rank-increasing properties of Krylov subspace methods based on partial commutativity of local operators in nearest neighbor Hamiltonians. In this talk, we will explain this idea and present numerical examples.